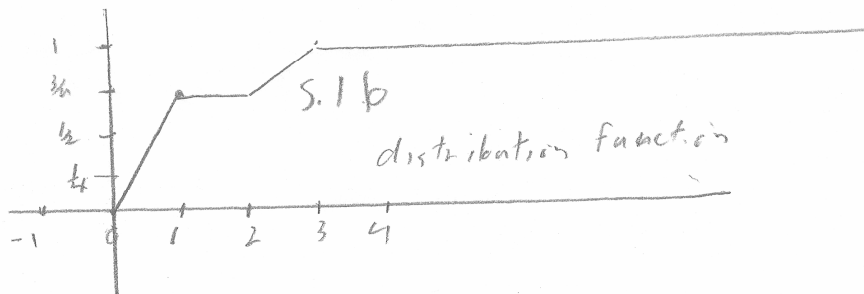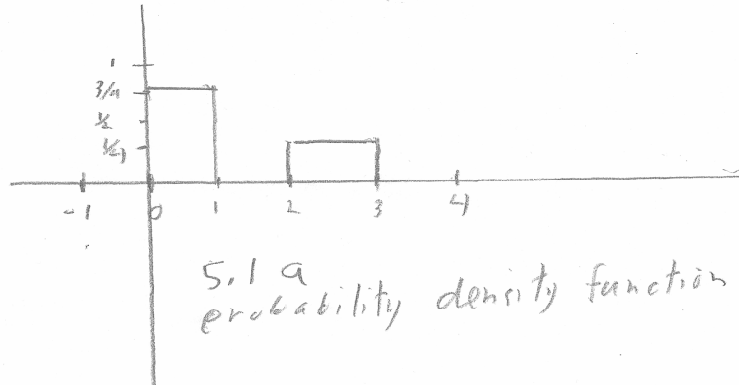Chapter 5 Problems

5.1

X is a continuous random variable with probability density function

f(x) =   0.75   if $0 \leq x \leq 1$
         0.25   if $2 \leq s \leq 3$
         0.0   elsewhere

5.1 a
probability density function

5.1 b
distribution function

5.5
Given a probability density function
f(x) = cx+3 if -3 ≤ x ≤ -2
      3-cx if 2 ≤ x ≤ 3
       0 elsewhere

a) Compute c.
The total area under f(x) must = 1.

The area between -3 and -2 is 3-c*5/2
The area between 2 and 3 is the same.
Therefore 6 – 5c = 1
5 = 5c or c = 1.

b) The distribution is found by integrating f(x)

$F(x) = 0$ for x < -3
     $x^2/2 + 3x$ for x in [-3, -2]
     0.5   for x in (-1, 2)
     $3x – x^2/2$ for x in [2, 3]
     1   for x > 3

5.9
a)
Let X = U(2,3) and Y = U(1,2) be uniformly distributed random variables.
Taking their Cartesian product gives us set of ordered pairs uniformly distributed within the rectangle in question. Let (x,y) be such an ordered pair. The area of the triangle determined by (2,1), (3,1), and (x,y) will be ½ the base times the altitude or
(3-2) * (y -1)/2 = (y-1)/2. Note that this is dependant only upon y and is unaffected by x.
As y goes from 1 to 2, A = (y-1)/2 goes from 0 to ½, so the area A is also a uniformly distributed random variable A = U(0, 0.5). The set of points (x, y) where
x is in [2, 3] and y is in [1, 1.25] are the points where A ≤ ¼.

5.11
The distribution function of Exp(λ) is F(a) = 1 – exp(-λa). The median is the value of a for which this = ½ (i. e. half of the probability is on either side of a).
1 - exp(-λa) = 0.5
0.5 = exp(-λa)
ln(0.5) = -λa
a = ln(0.5) / -λ

5.12

The distribution function for Par(1) is given by

F(x) = 1 – 1/x

The median is the value of x for which F(x) = ½.

1-1/x = 0.5
0.5 = 1/x
x = 2/